Experiment 2

DETERMINATION OF THE ACCELERATION DUE TO GRAVITY USING A SIMPLE PENDULUM

The aim of the experiment is to investigate the relation between the period of oscillation of a simple pendulum and its length and to determine the acceleration due to gravity.

INTRODUCTION

A simple pendulum consists of a point mass, m, suspended from a rigid support by a light string of length l as shown in figure 2.1.
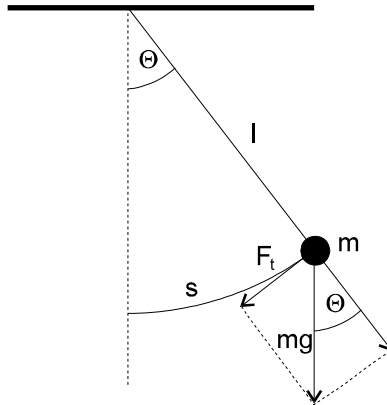

Fig. 2.1. Simple pendulum with a point mass m suspended from a support

Its oscillatory motion occurs in a vertical plane under the force of gravity, mg. The tangential component of the force, F_t is a restoring force acting in the direction opposite to the displacement, s, measured along the arc. It is equal to

$$F_t = -m\,g\sin\Theta \qquad\qquad (2.1)$$

For small Θ, $\sin\Theta \approx \dfrac{s}{l}$ and

$$F_t = - \, m \, g \, \frac{s}{l} \tag{2.2}$$

The force is proportional to the displacement s. The oscillation of a simple pendulum is then simple harmonic motion about the equilibrium position s=0. The period of motion is given by

$$T = 2 \, \pi \, \sqrt{\frac{l}{g}} \tag{2.3}$$

It depends only on the length of the string and the acceleration due to gravity. It is independent of the mass, m, and the displacement assuming this stays small.

APPARATUS AND METHOD

The simple pendulum can be used to measure the acceleration due to gravity which is equal to

$$g = 4 \, \pi^2 \, \frac{l}{T^2} \tag{2.4}$$

by measuring the period of oscillatory motion, T, for a given length, l. The pendulum contains an aluminum bob shaped to reduce damping by air and an unstretchable string. The string is fastened to a bobbin which allows the length of the string to be easily adjusted. The period of oscillation of the pendulum is found from the time of 20 complete swings taken against a mark fixed on the board. These measurements are repeated for different lengths of the pendulum determined from the point of suspension to the center of the bob. The reading must be rejected if the motion does not stay in the vertical plane.

The results of the measurements are plotted in a graph between the square of the period, T^2, and length, l, from which g can be obtained from the slope of the straight line. There is a linear dependence between T^2 and l according to

$$T^2 = \frac{4 \, \pi^2}{g} l \tag{2.5}$$

A typical graph obtained in the experiment is shown in figure 2.2. A straight line is drawn between the measured points and the acceleration due to gravity is calculated from the expression

$$g = 4 \, \pi^2 \, \frac{\delta l}{\delta T^2} \tag{2.6}$$

using the values of δl and δT^2 read from the figure. δl and δT^2 are the differences of the l and T^2 coordinates respectively calculated for two points lying on the straight line.
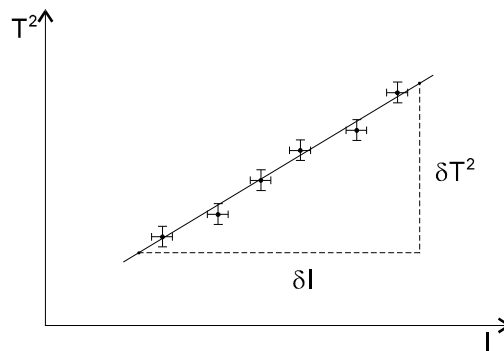

Fig. 2.2. A graph illustrating a linear dependence between the square of the period, T^2, and length, l, of the simple pendulum. δl and δT^2 are explained in the text

MEASUREMENTS

1. Measure the acceleration due to gravity using a simple pendulum. Repeat the measurements 5 times for a given length of pendulum and find a mean value of the period. Take the measurements for 5 different values of length of the pendulum in the range 0.9-1.5m. Keep the amplitude of the oscillations small (less than 6cm).
2. Verify experimentally that the period of oscillations is independent of the amplitude of swing providing this stays small.

ANALYSIS OR ERRORS

In the experiment errors in the measurements of the pendulum length, Δl, the period of oscillations, ΔT, the square of the period of oscillations, $\Delta(T^2)$, and the acceleration of gravity, Δg, have to be determined. Δl is the accuracy of the length measurement. ΔT is taken to be equal to the highest deviation of the measured period values from the mean value for each length l. These values of ΔT then are used to calculate $\Delta(T^2)$ error from the relation

$$\Delta(T^2) = 2\,T\,\Delta T \tag{2.7}$$

The $\Delta(T^2)$ and Δl errors are indicated in the graph by error bars for each point (figure 2.2) and allow the error of the acceleration of gravity Δg to be determined by a graphical method.

QUESTIONS

1. Explain the properties of simple harmonic motion.
2. How is the period of the pendulum effected when its point of suspension moves vertically upward with some acceleration ?
3. Will the frequency of oscillations of the pendulum change if it is taken to the moon ?
4. Explain why a pendulum oscillating with large amplitude will have a period longer than the pendulum oscillating with small amplitude.